Frege on Identity, Cognitive Value, and Subject Matter

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1   Introduction

No paragraph has been more important for the philosophy of language in the twentieth century than the first paragraph of Frege's 1892 essay "Uber Sinn und Bedeutung." He begins,

> Sameness [Gleichheit] gives rise to challenging questions which are not altogether easy to answer. Is it a relation? A relation between objects, or between names or signs of objects?[1]

Frege continues by explaining what bothered him in the *Begriffsschrift*, and motivated his treatment of identity in that work.[2] He goes on to criticize that account. By the end of the paragraph, he has introduced his key concept of *sinn*, abandonning not only the *Begriffsschrift* account of identity, but its basical semantical framework.

In the *Begriffsschrift* Frege's main semantic concept was *content* [*Inhalt*]. Already in the *Begriffsschrift*, he is struggling with this concept. In §3 he

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[1]Quotations from this paragraph use Black's translation ([Frege, 1960a], pp. 56-57) with a couple of changes, the most important of which is using "real" for both "eigentlich" and "wirklich", which I do to emphasize my view that a fairly clear and robust concept of "real knowledge" as opposed to knowledge about words and language plays a key role in Frege's argument. Black uses "proper" for "eigentlich" and "actual" for "wirklich".

[2]Henceforth I simply assume that it is identity, in the sense in which if A and B are identical there is only one thing that both is A and is B, that is at issue. See Frege's footnote.

emphasizes that the content of sentences that have different subjects and predicates can be the same; the subject/predicate distinction pertains to language, and not to content. In §9 he emphasizes that the function/argument structure we take a sentence to have represents a certain point of view. "Cato killed Cato" can be thought having "to kill Cato" or "to be killed by Cato" or "to kill oneself" as the function:

> For us the fact that there are various ways in which the same conceptual content can be regarded as a function of this or that argument has no importance so long as function and argument are completely determinate.([Frege, 1967], p. 23)

These remarks point to a concept of relatively unstructured contents. They are in line with the "semantic holism" that some of Frege's remarks and ideas have inspired.

In §8, "Identity of Content," however, a somewhat different view of content emerges. Individual signs that are parts of sentences have contents, and the combinations into which the signs enter express a relation among those contents — except in the case of identity sentences. The things Frege says about content in §8, setting aside the exceptional case, fit the following picture pretty well. Signs have contents, the things they stand for, among which are objects, functions, and relations. So the content of "2" is the number 2, the content of "+" the function of addition, the content of "<" the relation of smaller than, and so forth. The content of a complex will be determined by the contents of the parts, so the content of "2+3", namely 5, will be determined by the contents of "2", "3" and "+". The content of "$2 + 3 > 4$" will be determined by the contents of its parts, viz., the numbers 5 and 4, and the relation of larger than. Given this picture, the content of a sentence, the sort of thing that could be judged true or false, could be represented and individuated by a suitably structured complex of the contents of its parts.

The remarks we noted earlier do not fit this picture of structured content very well. We saw that Frege emphasizes that sentences with different functions and relations can have the same content: "$2 + 3 > 4$" and "$4 < 2 + 3$", for example. The idea seems to be that contents are individuated by unstructured truth conditions, so that two quite different structures could correspond to the same content, and sentential contents cannot be seen as complexes of the contents of their parts. Even in §9, however, following the remark quoted above, Frege seems to retreat to a more structured view when what he calls *indeterminate* arguments or functions are involved:

> ...if the argument becomes *indeterminate*, as in the judgement "You can take as argument of 'being representable as the sum of four squares' an arbitrary positive integer, and the proposition will always be true", then the distinction between function and argument takes on a *substantive* [inhaltliche] significance. On the other hand, it may also be that the argument is determinate and the function indeterminate. In both cases, through the opposition between the *determinate* and the *indeterminate* or that between the *more* and the *less* determinate, the whole is decomposed into *function* and *argument* according to its content and not merely according to the point of view adopted. ([Frege, 1967], p. 23)

These two pictures of content are in tension in Frege's early thought, and suffer different fates in his later thought [Weitzman, 1989]. For this discussion I am going to suppose that it is the structured contents, or perhaps the structured aspect of contents, that gave Frege a problem with identity in the *Begriffsschrift*, and led him to provide a solution there he later found unsatisfactory. I'll take the content of a statement $\phi(\alpha)$ to be, or at least to be representable as, a complex consisting of the condition ϕ designates and the entity α designates. If α designates an object, the conceptual content of $\phi(\alpha)$ will be similar to what we now call a *singular proposition*. If it

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designated a concept, it would be similar to what we now called a general or qualitative proposition.[3]

At the beginning of §8 Frege says,

> Identity of content differs from conditionality and negation in that it applies to names and not to contents. Whereas in other contexts signs are merely representatives of their content, so that every combination into which they enter expresses on a relation betweeen their respective contents, they suddenly display their own selves when they are combined by means of the sign for identity of content; for it expresses the circumstance that two names have the same content. ([Frege, 1967], pp. 20-21)

In this passage Frege expresses an account of content corresponding to the picture above, and then amends this theory to handle the problem about identity, so that a complex consisting of names and the relation of co-designation, rather than one consisting of object(s) and the relation of identity, is signified. The key semantic concept of the amended theory was still content; the effect of the amendment was simply to give identity statements a different content than they would have had without it.

In using the term "amended" I am suggesting that at some point Frege had a theory that didn't treat identity statements in a special way, then saw a problem, and amended that theory with special treatment of them that we find in the *Begriffsschrift*. Thus I am thinking of Frege's thought as falling into three periods, which I'll call "Ur-Frege", "Early Frege" and "Classical Frege". Classical Frege is the Frege of the "Über Sinn und Bedeutung". "Early Frege" is the Frege of the *Begriffsschrift*. And "Ur-Frege" is the hypothetical original period, before he decided to treat identity statements differently than other statements in the *Begriffsschrift*.

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[3]See [Kaplan, 1989a].

Now for all I know Frege's *Begriffsschrift* theory never had any unamended form. In using the terms "amendment", "amended theory", "unamended theory", and "Ur-Frege" as I do, I am simply adopting a way of referring to the theory he actually gave us (the theory "as amended") as one which *could* have resulted from amending a simpler theory (the "unamended theory") which had no special treatment of identity. I am not claiming that there was some earlier draft of the *Begriffsschrift* without the special treatment of identity. I am pretending that, for some period of time, perhaps no more than a few minutes, Frege had settled on the main outlines of the *Begriffsschrift* theory of content, but hadn't yet seen the need, or what he took to be the need, for a special treatment of identity statements.

Classical Frege criticizes the *Begriffsschrift* theory, and introduces the concept of sinn and the distinction between sinn and bedeutung not only as an alternative to the *Begriffsschrift* way of handling the problem, but as a new semantical framework.[4] Sinn and bedeutung replace content in Frege's thinking; sinne are more fine-grained than contents, and bedeutungen are less fine-grained. Frege's level of bedeutung was the inspiration for the semantic values we now think of as extensions: objects, sets of objects, and truth-values. Extensions provide the standard interpretation of the predicate calculus, and, according to Quine constitute the only intelligible semantic values for a scientifically respectable language. Frege's level of sinn was the inspiration, at least in part, for much of the work on intensional logic, from Church and Carnap to Kripke and Kaplan.[5]

Thus the argument in the first paragraph is important, if for no other reason, because it records an important part of the motivation for Frege's theory of sinn and bedeutung. Some think that Frege's argument in this paragraph

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[4]To avoid problems of translation, I use "sinn" and "bedeutung" as English words, with the plurals "sinne" and "bedeutungen".

[5]See [Church, 1956], [Carnap, 1956], [Kaplan, 1989a], [Kripke, 1963]. In contrast Jaakko Hintikka's model for intensions was *multiple reference* rather than sinne [Hintikka, 1969].

gave him a valid reason to abandon the content framework—especially the structured version of it—in favor of the sinn and bedeutung framework. This is a mistake. The considerations Frege raises in this paragraph, including his criticism of the *Begriffsschrift*, provide no reason whatsovever for abandoning structured content as his basic semantic concept.

On my view, Frege's considerations provide instead reasons for seeing both the content provided by a sentence and the cognitive value of a sentence as relative concepts. The concept of "real knowledge" is motivated the distinction between *linguistic* conventions that is presupposed in the use of language, and the knowledge about the things the linguistic items designate, which is what we use senteces to record, store and communicate information about. But what counts as presupposed knowledge of language, and what counts as real knowledge, is not fixed for all situations. Consider, for example, these two sentences

The biggest city in Nebraska = the biggest city in Nebraska.

The biggest city in Nebraska = the municipality larger than any other in Nebraska.

Both statements are reasonably thought of as guaranteed to be true by the conventions of English, and so analytic. But the conventions at issue are quite different. With the first sentence, knowledge of the convention that sentences of the form $A = A$ are true will suffice.[6] With the second sentence, knowledge of the meaning of the various words is also required. The different sorts of knowledge required to recognize the analyticity corresponds to different types and subsets of the conventions of English that account for the analyticity. The second sentence might be used to convey to a person the meaning of the word "municipality". One would be exploiting the person's

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[6]As a statement about natural language, this would need to be qualified in various ways, which I ignore.

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knowledge of many of the conventions of English, in order to convey the information that that "municipality" means about the same as "city".

I argue that if we appreciate the relativity of both the information contained in a sentence and the cognitive value of a sentence, Frege's considerations need not push us away from the view of structured content suggested in §8 of the *Begriffsschrift*.

So, in this paper, I provide an interpretation of Frege's opening paragraph, and then argue that (i) Classical Frege is correct that the *Begriffsschrift* account is flawed; (ii) However, the *Begriffsschrift without the amendment*, could deal successfully with the problem. Thus, whatever strong arguments there may be for abandoning the content framework in favor of the sinn and bedeutung framework, they are not to be found in Frege's treatment of the problem of informational identities in his opening paragraph.

2 Some interpretive tools

2.1 Propositions

I'll assume that any n-ary condition R and sequence $< e_1 \ldots e_n >$ of appropriate entities determine a $\mathcal{B}$-*proposition* $P = \mathcal{B}(R, e_1 \ldots e_n)$, which is true if and only if $< e_1 \ldots e_n >$ meet the condition R. I assume that conditions have parameters or argument roles that can be ordered so that we can talk about the *i-th* argument role of the condition R. I assume that each parameter is suitable for either an object or a condition. If the *i-th* parameter of R is suitable for an object I will say that P is *objectual* at its *i-th* parameter. If it is suitable for a condition, I will say that P is *qualitative* at its *i-th* parameter. A proposition that it qualitative at all of its parameters is a *general* proposition, one that is objectual at some parameter is a *singular* proposition.

$\mathcal{B}$-propositions are intended only to enable us to model certain choices that Frege had within the framework of structured content, as conceived

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according to the above picture. Thus I assume only that the contents of *simple* statements are *something like* $\mathcal{B}$-propositions.

2.2 The example

Frege gives rather similar examples of identity statements in the *Begriffss-chrift* and *Über Sinn und Bedeutung*. Although he ends up with two different accounts in the two works, in both cases the examples force us to distinguish between a single point and two modes of presentation of that point. I want to use just one example to compare the two treatments, so I'll use the simpler and more familiar example from *Über Sinn und Bedeutung*. It involves three lines, a, b, and c, that intersect at the same point. The informative identity statement is:

(1) The point of intersection of a and b = the point of intersection of b and c.

Call the term on the left hand side of (1) α and the term on the right hand side β. Call the point they both designate, "Albert". The uninformative identity statement is:

(2) The point of intersection of a and b = the point of intersection of a and b.

2.3 Some key concepts

Frege's two treatments of identity are both motivated by a problem that involves the following concepts. All of these concepts are more or less explicit in "Über Sinn und Bedeutung".

- The **subject matter** of a sentence. This is the objects (or conditions) designated by the terms in the sentence, and the condition designated by the condition word in the sentence. The condition is asserted of the objects for which the terms stand. Take for example the sentence

"α is inside a circle". The subject matter is the point Albert, which is designated by α, and the condition of being inside a circle.

- The **Epistemic value** [*Erkenntniswert*] of the sentence. As far as the cases discussed in the first paragraph, this means being a priori and analytic, or a posteriori and synthetic. $a = b$ is assumed to be an example of a synthetic sentence, while $a = a$ is a priori.

- The **real knowledge** [*eigentliche Erkenntnis, wirkliche Erkenntnis*] expressed by the sentence. This determines the cognitive value of the sentence. It is not knowledge about signs, but knowledge *above and beyond that contained in the knowledge of the linguistic conventions*; hence, knowledge that can be conveyed by different languages.

I'll call a condition (a relation or property) an *identifying condition* if it is the sort of condition that at most one thing can meet. That is, an identifying condition is the sort of condition that we express with the definite article, like "x is the most influential German philosopher of the nineteenth century" or "x is the point of intersection of a and b". I use two special kinds of identifying conditions and the relation of co-instantiation among conditions to reconstruct Frege's argument. Where Fx and Gx are conditions there will be a qualitative proposition that there is something that co-instantiates them, which I'll call "CI". So $\mathcal{B}(CI; Fx, Gx)$ is the $\mathcal{B}$-proposition that Fx and Gx are coninstantiated.

- **Modes of designation**. I take a mode of designation to be a condition of being designated by a certain term, e.g., being the designatum of "Gottlob Frege". A mode of designation is hence a species of an identifying condition, a condition that only one thing may satisfy. Consider:

  $\mathcal{B}(CI; x$ is the designatum of "Gottlob Frege", x was a logician)

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This is the proposition whose condition is CI, the relation that holds between two conditions when some object instantiates both. The first condition is a mode of designation, a condition that will be satisfied by at most one thing, the thing that is designated by "Gottlob Frege", namely, Frege. The second condition is the condition of having been a logician. This proposition is true, since there is one thing, the person Frege, that satisfies the mode of designation, and he also satisfies the second condition, of having been a logician.

- **Modes of presentation**. A mode of presentation is also an identifying condition. But a mode of presentation has to do with satisfying a condition expressed by the words, not a condition relating directly to the words. It is one thing to be designated by the words "the present Queen of England" and another thing to be the present Queen of England. The first condition is a mode of designation that Elizabeth II satisfies, the second a mode of presentation that she satisfies. It is one thing to be designated by the words "the point of intersection of a and b" and another to be the point of intersection of a and b. The point we named "Albert" satisfies both.

If we consider a sentence like "The point of intersection of a and b is inside of a circle" we can distinguish among three propositions that will have to be true for the sentence to be true.[7] First, there is $\mathcal{B}(x$ is inside a circle; Albert),

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[7]I assume that in order to be the object designated by "the ϕ", an object has to have the property designated by ϕ. In Donnellan's famous example, a speaker successfully uses the phrase "the man drinking a martini" to point out a man who is drinking a glass of water with an olive [Donnellan, 1966]. He uses this as an example of a "referential" as opposed to an "attributive" use of a definite description. The points I am making don't hold for that kind of example, and I don't here discuss how to treat such cases. Donnellan's concept of "referential" combines the feature of allowing inaccuracy, as in the martini case, and the feature of conveying a singular proposition. I do not need to assume that the description is used attributively, in the sense of expressing a general proposition as opposed to a singular one. It may be used referentially, in the sense of expressing a singular proposition about whoever is in fact the ϕ. As long as to be the object the proposition is about the object must be the ϕ, as opposed to merely almost being the ϕ, or being taken by speaker or listener to be the ϕ, the points I am making apply.

a singular proposition, the **subject matter proposition**.

Then there is $\mathcal{B}(CI; x$ is the object designated by α, x is inside a circle). This is a general proposition, which I call the **modes of designation proposition**.

Finally there is $\mathcal{B}(CI; x$ is the point of intersection of a and b, x is inside a circle). This is a general proposition, which I call the **modes of presentation proposition**.

2.4 The Problem

Let $A = B$ and $A = A$ be forms of statements, and $\alpha = \beta$ and $\alpha = \alpha$ be particular statements of those forms. Suppose that $\alpha = \beta$ is true, and that there is no internal connections of structure or dictionary meaning that insure that α and β designate the same thing. Now consider these six statements:

(A) (i) $\alpha = \beta$ is true

    (ii) $\alpha = \beta$ is not a priori

    (iii) $\alpha = \alpha$ is a priori.

(B) A statement of the form $A = B$ asserts identity of the designata of A and B, that is, its subject matter proposition is $\mathcal{B}($ Identity; $A, B)$.

(C) Hence, $\alpha = \alpha$ and $\alpha = \beta$ have the same subject matter proposition (from (A(i)) and (B)).

(D) The real knowledge expressed by a sentence is its subject matter proposition.

(E) The real knowledge expressed by a sentence determines its cognitive value.

(F) Hence, $\alpha = \beta$ and $\alpha = \alpha$ have the same cognitive value (from (C), (D), and (E)).

But, clearly,

(G) $\alpha = \alpha$ and $\alpha = \beta$ have different cognitive values. (From (A)(ii) and (A)(iii))

This is my reconstruction of the problem that bothered Frege. In the *Begriffsschrift*, he resolves it by giving up (B). In *Über Sinn und Bedeutung* he resolves it by giving up (D). I will argue that (D) and (E) both need modification.

3 Frege's *Begriffsschrift* theory

Now let's return to the text. Frege asks, then, whether sameness, if it is a relation, is a relation between objects, or between names or signs of objects. He continues:

> In my *Begriffsschrift* I assumed the latter.

Let's take another look at relevant passage in his *Begriffsschrift* to remind ourselves what he actually said there.

> Identity of content differs from conditionality and negation in that it applies to names and not to contents. Whereas in other contexts signs are merely representatives of their content, so that every combination into which they enter expresses on a relation betweeen their respective contents, they suddenly display their own selves when they are combined by means of the sign for identity of content; for it expresses the circumstance that two names have the same content. Hence the introduction of a sign for identity of content necessarily produces a bifurcation in the meaning of all signs: they stand at times for their content, at times for themselves. ([Frege, 1967], pp. 20-21)

Almost seeming to anticipate the objection he will make in "Über Sinn und bedeutung", he goes on,

> At first we have the impression that what we are dealing with pertains merely to the *expression* and *not to the thought...* ([Frege, 1967], p. 21)

He rebuts this suggestion, as resting on the false premiss that

> ...we do not need different signs at all for the same content and hence no sign whatsoever for identity of content. ([Frege, 1967], p. 21)

To show that this is wrong-headed, Frege uses an example that is quite similar to, although as I mentioned, more complicated than, the one he was to use in "Über Sinn und Bedeutung". For simplicity, I am pretending that he simply uses the later example in both places. So the informative identity statement is:

(1) The point of intersection of a and b = the point of intersection of b and c.

We are calling the term on the left hand side of (1) α and the term on the right hand side β. Frege points out—in the *Begriffsschrift* !— that α and β correspond to two ways of determining the content. He then says:

> To each of these ways of determining the point there corresponds a particular name. Hence the need for a sign of identity rests upon the following consideration: the same content can be completely determined in different ways; but that in a particular case *two ways of determining it* really yield the *same result* is the content of a *judgment*. ([Frege, 1967], p. 21)

There is a certain tension in this account. There seem to be three possible $\mathcal{B}$-propositions one might want to associate with (1).

(2) $\mathcal{B}($ Identity; Albert, Albert): the singular proposition about Albert, to the effect that it is identical with itself. This is the subject matter proposition on the unamended theory, that treats identity statements like everything else. Frege clearly doesn't want this.

(3) $\mathcal{B}(CI; \alpha$ designates x, β designates $x)$: the proposition that the two modes of designation are coinstantiated. This is (roughly) Early Frege's candidate for the subject matter proposition.

(4) $\mathcal{B}(CI; x$ is the point of intersection of a and b, x is the point of intersection of b and c): the proposition that the two modes of presentation are coinstantiated. This is not the content according to Early Frege, although at the end of the passage I quoted above, it sounds as if it might be what he really wanted. At any rate, this is more or less the proposition that Classical Frege came to think was the "real knowledge" conveyed by (1).

4 Frege's Reconstruction

So now to return to the first paragraph of "Über Sinn und Bedeutung". So far Frege has said, to repeat,

> Sameness gives rise to challenging questions which are not altogether easy to answer. Is it a relation? A relation between objects or between names or signs of objects? In my *Begriffsschrift* I assumed the latter.

He then goes on to give the reasons which "seemed" to favor this:

> The reasons which seemed to favor this are the following: $a = a$ and $a = b$ are obviously statements of differing epistemic value [Erkenntniswert]; $a = a$ holds *a priori* and, according to Kant is to be labelled analytic, while statements of the form $a = b$ often

contain very valuable extensions of our knowledge and cannot always be established *a priori*. The discovery that a new sun does not rise every morning, but always the same one, was one of the most fertile astronomical discoveries. Even today the identification of a small planet or a comet is not always a matter of course.

Frege presumably means to recall the considerations that found expression in the *Begriffsschrift* simply as,

The same content can be fully determined in different ways.

In the *Begriffsschrift*, he called the object which a name designated the *content* of the name. And as we saw in the *Begriffsschrift* he really seems to have in mind, when he talks about "two ways of determining," not simply two modes of designation or two signs, but two modes of presentation.

He then says,

Now if we were to regard sameness as a relation between that which the names 'a' and 'b' designate, it would seem that $a = b$ could not differ from $a = a$ (i.e. provided that $a = b$ is true). A relation would thereby be expressed of a thing to itself, and indeed one in which each thing stands to itself but to no other thing.

Above I reconstructed the problem that bothered Frege in terms of (A) – (G). Now I'll use that reconstruction to explain Classical Frege's reconstruction of Early Frege:

Suppose:

(B) A statement of the form $A = B$ asserts identity of the designata of A and B, that is, its subject matter proposition is $\mathcal{B}(\text{Identity}; A, B)$.

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and

(A i) $\alpha = \beta$ is true

Then:

(C) $\alpha = \alpha$ and $\alpha = \beta$ have the same subject matter proposition.

Then:

(F) $\alpha = \beta$ and $\alpha = \alpha$ have the same cognitive value

But we saw above:

(A ii) $\alpha = \beta$ is not a priori

(A iii) $\alpha = \alpha$ is a priori.

and thus

(G) $\alpha = \alpha$ and $\alpha = \beta$ have different cognitive values.

So our supposition must be wrong—we must reject (B).

This conclusion corresponds to only half of the position in the *Begriffss-chrift*, the half that says that identity statements do *not* assert identity of the designata of their terms. Classical Frege provides an argument for the second half of Early Frege's position, that identity statements *do* assert co-designation of the terms, in the next passage:

> What is intended to be said by $a = b$ seems to be that the signs or names 'a' and 'b' designate the same thing, so that those signs themselves would be under discussion; a relation between them would be asserted.

For future reference, I want the reader to focus on Frege's thought here. He is reporting, accurately, that at the time of *Begriffsschrift*, he had noted that "a=b", unlike "a=a", carries the information that the signs "a" and "b" co-designate. This seems quite correct. Suppose I am puzzled, driving through the Basque Country, by the fact that the mileage signs always have Donastia and San Sebastian exactly the same distance away. Seems an odd coincidence. You tell me that "Donastia is San Sebastian". One thing I will learn, whatever else I may learn, is that "Donastia" and "San Sebastian" name the same city. Early Frege explained this by holding this bit of information is what you asserted. Classical Frege gives good reasons for thinking this is not what is asserted. But he does not give reasons for denying what Early Frege noticed, that this information is conveyed.

5 Frege's critique of his earlier view

At this point, the word "But" signals the shift back to Classical Frege's own opinions:

> But this relation would hold between names or signs only insofar as they named or designated something. It would be mediated by the connection of each of the two signs with the same designated thing. But this is arbitrary. Nobody can be forbidden to use any arbitrarily producible event or object as a sign for something. In that case $a = b$ would no longer refer to the subject matter, but only to its mode of designation; we would express no real knowledge by its means. But in many cases this is just what we want to do.

Here Classical Frege seems to be zeroing in on just the tension we noted in the *Begriffsschrift* discussion. Note that he is not directly rebutting the negative argument, the argument that sameness sentences do *not* express a relation between the objects designated by their terms.

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The term "real knowledge" ["eigentliche Erkenntnis"]," bears considerable weight.

Recall (1):

(1) The point of intersection of a and b = the point of intersection of b and c.

We noted that in order for (1) to be true, three quite different propositions must be true. The first is about Albert, the designatum of the terms flanking the identity sign; Albert must be identical with Albert. The second is about the modes of designation, that they are coinstantiated. The third is about modes of presentation, that they are coinstantiated.

Which of these three is the content of (1)? In the passage just quoted, Classical Frege provides a reason to favor the mode of presentations proposition (his choice) over the mode of designations proposition (Early Frege's choice). The reason is that the knowledge that is embodied in the modes of designations proposition is not *real* knowledge, but knowledge that a certain relation holds between signs.

As I interpret the next passage,

> If the sign "a" is distinguished from the sign "b" only as object (here, by means of its shape), not as sign (i.e. not by the manner in which it designates something), the cognitive value of $a = a$ becomes essentially equal to that of $a = b$, provided that $a = b$ is true.

Frege tries, not completely successfully, to provide a reason for favoring the modes of presentations proposition over the subject matter proposition, that Albert is Albert. The basic idea is that the real knowledge, which determines the cognitive value, is what we learn in addition to what is fixed by our knowledge of language; that is, what the truth of the sentence requires beyond what is required by the truth of the conventions of language. To

consider signs "only as objects" is to ignore the fact that the conventions of language connect the signs with modes of presentation, not directly with objects. If we disregard this link between the signs and modes of presentation, and consider only the link between signs as differently shaped objects and their designata, then the conventions of language determine the truths of both sentences. Language assigns the terms a and b to the same object, and so $a = b$, like $a = a$, doesn't provide us with any information beyond the conventions of language.

This passage seems odd, because even if we ignore the manner in which signs designate, there is still a difference between $a = a$ and $a = b$; the shapes that flank the identity sign are the same in the first case but are not the same in the second. The convention that different occurrences of the same shape designate the same thing would assure the truth of $a = a$, but not of $a = b$. So what is Frege driving at? On my interpretation he is driving at the fact that if we ignore modes of presentation, our semantical rules will directly tie signs, considered simply as objects of various shape, to their designata. In that case, and given that it is true, the truth of $a = b$ will be determined by the semantical rules. And then its cognitive value will be the same as $a = a$—analytic and a priori. He is basically arguing against a view of reference or designation as unmediated by a mode of presentation, and hence, almost a century in advance, against some versions of the "direct reference" theory.[8]

Classical Frege then seems to be eliminating possibilities to arrive at the right view, as follows:

- The mode of designations proposition is not analytic, and constitutes knowledge, but it is not *real* knowledge, but knowledge about signs. So, contrary to the *Begriffsschrift*, it won't do.

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[8]I say "some versions" of direct reference theory, because the definition of "direct reference" that Kaplan gives in [Kaplan, 1989a], where the term is introduced into the literature, doesn't reqire that the mechanism of reference be unmediated. See [Marti, 1995] and [Perry, 1997].

- The subject-matter propositions aren't about language. But they are the same for both sentences. What's more, the point of view that leads to this choice seems to make the truth of both a matter of the conventions of language. So sentences of the forms $a = b$ and $a = a$ would have the same content, and both be a priori, if we were to make this choice for their content.

So the subject matter proposition doesn't give us *a posteriori* knowledge, the mode of designations proposition doesn't give us *real* knowledge, and that leaves the mode of presentations proposition:

A difference can arise only if the difference between the signs corresponds to a difference in the mode of presentation of that which is designated. Let a, b, c be the lines connecting the vertices of a triangle with the midpoints of the opposite sides. The point of intersection of a and b is then the same as the points of intersection of b and c. So we have different designations for the same point, and these names ('point of intersection of a and b', 'point of intersection of b and c') likewise indicate the mode of presentation; and hence the statement contains real [wirkliche] knowledge.

This passage makes a very similar point to the one we quoted from the *Begriffsschrift*. There he noted that we needed two signs to convey the knowledge that is involved in this sort of example, that two modes of presentation are of the same object. Here he twists the same point around: two names flanking an identity sign would not be cognitively different from one another, at least as far as real knowledge goes, unless they were associated with different modes of presentation.

6   Classical Frege's Position

In terms of (A)-(G), here is what I take to be Classical Frege's position.

He accepts (A), (E) and (G).

(A) (i) $\alpha = \beta$ is true

(ii) $\alpha = \beta$ is not a priori

(iii) $\alpha = \alpha$ is a priori.

(E) The real knowledge expressed by a sentence determines its cognitive value.

(G) $\alpha = \alpha$ and $\alpha = \beta$ have different cognitive values. (From (A)(ii) and (A)(iii)).

He rejects (D), and replaces it with (D'):

(D) The real knowledge expressed by a sentence is its subject matter proposition.

(D$'$) The real knowledge expressed by a sentence is its modes of presentation proposition.

And thus avoids (F) and the contradiction with (G):

(F) $\alpha = \beta$ and $\alpha = \alpha$ have the same cognitive value.

The concept of subject matter comes to a dead end in Frege's philosophy at this point. He retains the concept of the bedeutungen of the parts of a statement, and of a statement being about these bedeutungen, but the concept of a subject matter proposition, a complex that embodies the claim the statement makes about the bedeutungen of its terms, vanishes. In its place is only the truth-value, at the level of bedeutung, and the thought expressed by the statement, at the level of sinn. (B) and (C)

(B) A statement of the form $A + B$ asserts identity of the designata of a and b, that is, its subject matter proposition is $\mathcal{B}($ Identity$; A, B)$.

(C) $\alpha = \alpha$ and $\alpha = \beta$ have the same subject matter proposition (from (A(i)) and (B)).

play no role in Frege's solution, but he also gives us no reason, in this paragraph, to reject them as false. They simply drop from sight, because the key concept in them, the subject matter proposition, plays no more role in Frege's semantics.

7   Back to Ur-Frege?

The solution to the problem of identity and epistemic significance that Frege provides in the first paragraph is completely compatible with the Ur-Frege account and the semantics of structured content. He has provided a reason for abandoning the *Begriffsschrift* treatment of identity sentences, but not for abandoning the *Begriffsschrift* account of content.

A semantically competent user of the relevant language who hears (1), believes the speaker, and has a clear view of the diagram, will learn *all three* of the propositions in question, the subject matter proposition, the mode of designations proposition, and the mode of presentations proposition. All three propositions must be true, if (1) is to be true; each of them is in that sense part of the truth conditions of (1). The mode of designations proposition must be true, if (1) is to be true, *given* that (1) is an English statement of the form $a = b$. The mode of presentations proposition must be true, *given* all of that, *plus* the fact that "the point of intersection of a and b" and "the point of intersection of b and c" mean what they do. And the subject matter proposition must be true, *given* all of *that*, and adding the fact that Albert is the point of intersection of a and b and the point of intersection of b and c.

Note that credulous hearer who *wasn't* a fully competent speaker or who didn't hear the words clearly might learn the mode of designations proposition; this might help him to identify the words, or to learn their

meanings. Someone who understood the language, but couldn't see the diagram, and identify the referent of the terms, might learn the mode of designations and the mode of presentations propositions, but not the subject matter proposition.

The Ur-Frege theory gives us all the semantics we need to distinguish the three propositions, and to account for the fact that all three must be true for (1) to be true.

What about the cognitive value of (1)? Frege's view that (1) is not analytic and a priori, but synthetic and a posteriori, in virtue of the different meanings of the terms, seems quite reasonable. That is to say that it is the truth conditions of a statement, fixing only the facts about meaning, and not those about reference, in which we are interested when we ask whether something is analytic or a priori.

But is that the right way to look at "a=a"? We don't have to look at the modes of presentation associated with the terms of this identity, to know that it was true. The meaning of "=" and the conventions (of a language as well-behaved as Frege wanted his *Begriffsschrift* to be) that terms name something and that the same terms name the same thing guarantee the truth, without taking into account the conventions that assign meaning to "a".

A reasonable way to look at it seems to be that a statement can be analytic in virtue of *various* subsets of the conventions of language that pertain to it. Analyticity is relative to which aspects of meaning are kept fixed. To return to the example I introduced earlier, two analytic statements can differ in as the following pair do: "The biggest city in Nebraska is the biggest city in Nebraska" and "The biggest city in Nebraska is the municipality larger than any other in Nebraska." A person who knew English fairly well, but didn't know what "municipality" meant, might know the first to be true in virtue of meaning but not the second. They are analytic in virtue of *different* conventions of language. It is this relativity that Frege needed

in this ur-theory, in addition to structured content, to handle the problem of identity, not a special treatment of identity sentences, and not the new semantical approach of "Über Sinn und Bedeutung".

I conclude that a theory of analyticity, and the informational value of statements, based on the Ur-Frege semantic apparatus of structured content, can support a theory that handles the examples Frege considers in the first paragraph of "Über Sinn und Bedeutung", in line with the points he makes there. The considerations of the first paragraph of this famous essay provide no reason for abandoning the basic semantic approach of the *Begriffsschrift*, but only for abandoning the special treatment of identity statements contained therein.

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